Nonlinear pulse propagation in optical fibers with randomly varying birefringence

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Abstract :

The Manakov PMD equation was derived from the Maxwell equations to study light propagation over long distances in optical fibers with randomly varying birefringence [1]. We denote by L_B the beat length and l_c the correlation length. The lengths l_{nl} and l_d are related respectively to the Kerr effect and to the chromatic dispersion. Considering the following regime $L_B \ll l_c \ll l_{nl} \sim l_d$, we introduce a dimensionless parameter $\epsilon > 0$, given by the ratio of these lengths. The slowly varying envelope X_ϵ has then the following evolution

$$i\frac{\partial X_\epsilon(z)}{\partial z} + \frac{ib'}{\epsilon}\boldsymbol{\sigma}\left(\nu_\epsilon(z)\right)\frac{\partial X_\epsilon(z)}{\partial t} + \frac{d_0}{2}\frac{\partial^2 X_\epsilon(z)}{\partial t^2} + F_{\nu_\epsilon(z)}(X_\epsilon(z)) = 0, \tag{1}$$

where ν is a stochastic process, d_0 is the group velocity dispersion, b' is the frequency derivative of the birefringence strength b and $F_{\nu_\epsilon(t)}(X_\epsilon(t))$ is a cubic nonlinearity.

In this talk, I will explain why the asymptotic dynamics (when ϵ goes to 0) is described by a stochastic perturbation of the Manakov equation [2,3]

$$i\partial_z X(z) + \frac{d_0}{2}\frac{\partial^2 X(z)}{\partial t^2} + F\left(X(z)\right) + i\sqrt{\gamma}\sum_{k=1}^{3}\sigma_k\frac{\partial X(z)}{\partial t}\dot{\xi}_k(z) = 0. \tag{2}$$

The positive constant γ is a small positive parameter given by the physics of the problem, $\dot{\xi} = (\dot{\xi}_1, \dot{\xi}_2, \dot{\xi}_3)$ is a white noise in time. The nonlinear interaction term in Equation (2) is given by $F\left(X(z)\right) = \frac{8}{9}\left|X\right|^2 X(z)$ and the nonlinear PMD effects have been averaged out to zero. I will also display numerical simulations of soliton's propagation subject to PMD and statistics of the PMD.

References :

1. P. K. A. Wai and C. R. Menyuk, *Polarization mode dispersion, decorrelation, and diffusion in optical fibers with randomly varying birefringence*, Journal of Lightwave Technology **14**, 148–157 (1996).
2. A. de Bouard and M. Gazeau, *A diffusion approximation theorem for a nonlinear PDE with application to random birefringent optical fibers.* Ann. Appl. Probab. **22**, 2460–2504 (2012).
3. M. Gazeau, *Analyse de modèles mathématiques pour la propagation de la lumière dans les fibres optiques en présence de biréfringence aléatoire*, Thèse de Doctorat, Ecole Polytechnique (2012).